Basics of Radio Interferometry

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Second International Convention
Of the ***European Radio Astronomy Club***
And the ***S.E.T.I. League USA***

Heppenheim, September 9-10, 2000

What will we talk about?

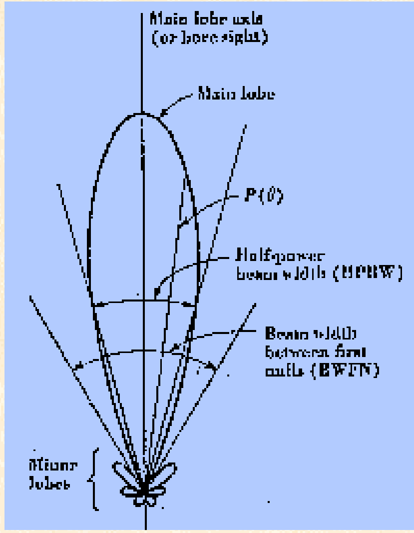
- Motivation for Radio Interferometry
- Basic Ideas
- The Two-Element Interferometer
  (basic interferometer equations)
- Aperture Synthesis
- Amateur Radio Astronomy and Interferometry

Motivation for Radio Interferometry

- angular resolution of a telescope $\propto \lambda/D$
- optical telescopes: 20 marcsec

  (D=5m, λ=500nm)
- radio telescopes: 1 arcmin

  (D=100m, λ=2.8cm)



- extra-galactic radio sources:

  fine scale structures < 1 marcsec

  (1marcsec @ λ = 2.8cm $\Rightarrow$ D = 6000km)
- filled aperture telescopes limited to D $\approx$ 100m

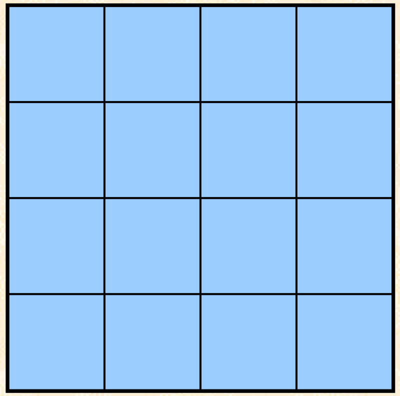
The Solution

- There **is** a way to build big radio telescopes:
  - take several “small” telescopes in great distance from one another
  - combine their output signals in an appropriate way
  - do some computing on the results

That is a very simplistic view of a **radio interferometer**

Basic ideas I

- fixed aperture antenna composed from N elemental areas
- each element n is contributing a signal
  $I_n \cos(\omega t + \Phi_n)$
- vectorial addition of all signals yields:



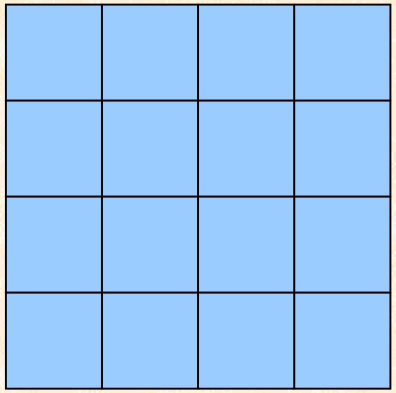
N = 16

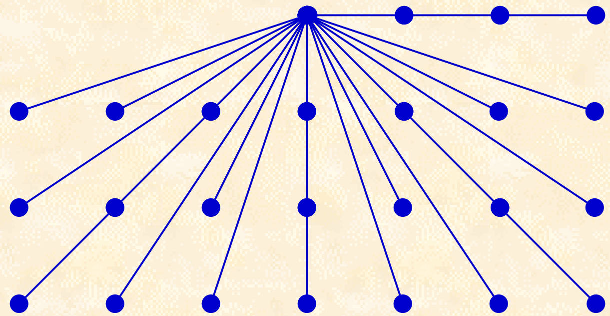
$$P \propto \tfrac{1}{2} \cdot \sum_{j=1}^{N} \sum_{k=1}^{N} I_j I_k \cos(\Phi_j - \Phi_k)$$

$$= \tfrac{1}{2} \cdot \sum_{j=1}^{N} I_j^2 + \sum_{j=1}^{N-1} \sum_{k=j+1}^{N} I_j I_k \cos(\Phi_j - \Phi_k)$$

Basic Ideas II

- cross terms can be measured separately one pair at a time
- addition to be done later
- two moveable antennas → simulation of a large dish



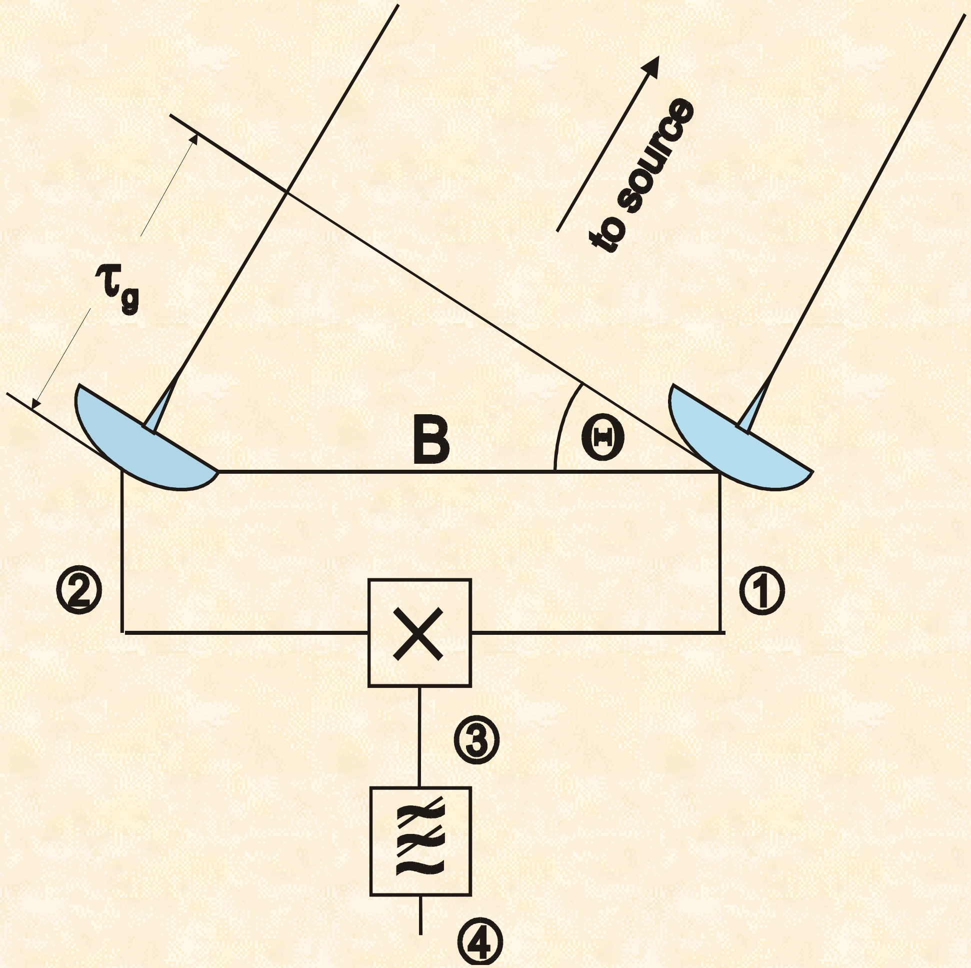


The Two-Element Interferometer I

B = Baseline

Θ = angle between base line and wave front from source

$\tau_g = B \cdot \sin\Theta / c$ wave propagation (geometric) delay



The Two-Element Interferometer II

- assuming a point source and monochromatic radiation:
  - ① $R_1(t) = E \cdot \cos \omega t$
  - ② $R_2(t) = E \cdot \cos \omega(t - \tau_g)$
  - ③ $R_3(t) = R_1(t) \cdot R_2(t)$

$$= [E \cdot \cos \omega t] \cdot [E \cdot \cos \omega(t - \tau_g)]$$

$$= \tfrac{1}{2}E^2 \cdot [\cos \omega(t + t - \tau_g) + \cos \omega(t - t + \tau_g)]$$

$$= \tfrac{1}{2}E^2 \cdot [\cancel{\cos \omega(2t - \tau_g)} + \cos \omega\tau_g]$$

  - ④ $R_4 = \tfrac{1}{2}E^2 \cdot \cos \omega\tau_g$

The Two-Element Interferometer III

- tracking the source / earth rotation make:

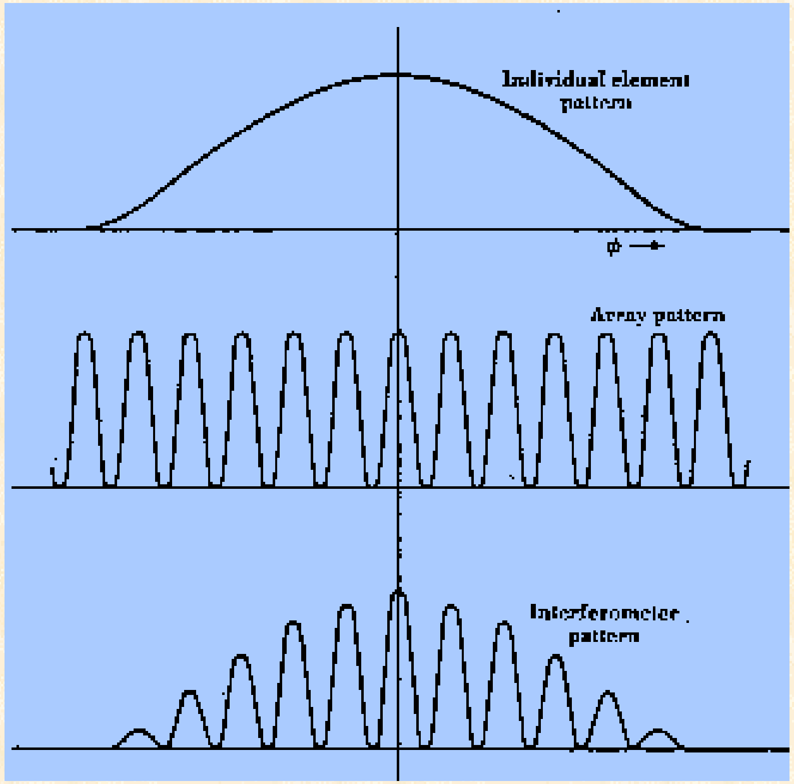
  $\Theta = \Theta(t)$ and $\tau_g = \tau_g(t) = B \cdot \sin \Theta(t) / c$

- therefore

$$R_4(t) = \frac{1}{2}E^2[\cos(\omega \frac{B \cdot \sin\Theta(t)}{c})]$$

$$= \frac{1}{2}E^2\left[\cos\left(\frac{2\pi B \cdot \sin\Theta(t)}{\lambda}\right)\right] \approx \frac{1}{2}E^2 \cdot \cos\left(\frac{2\pi B \cdot \Theta(t)}{\lambda}\right)$$

The Two-Element Interferometer IV



←single antenna characteristic

←tracking the source

←transit instrument

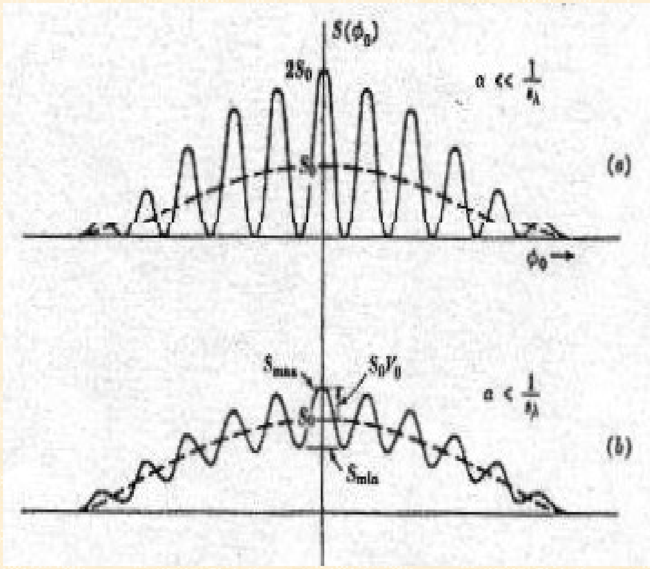
Requirements for a Working Interferometer

- Technical requirements:
  - local oscillators for mixers of both telescopes phase-locked
  - RF lines from antennas to receivers of equal length

- Radiation requirements
  - planar wave fronts
  - coherence length » $\tau_g \cdot c$
  - variation of radiation intensity slow when compared to τ_g

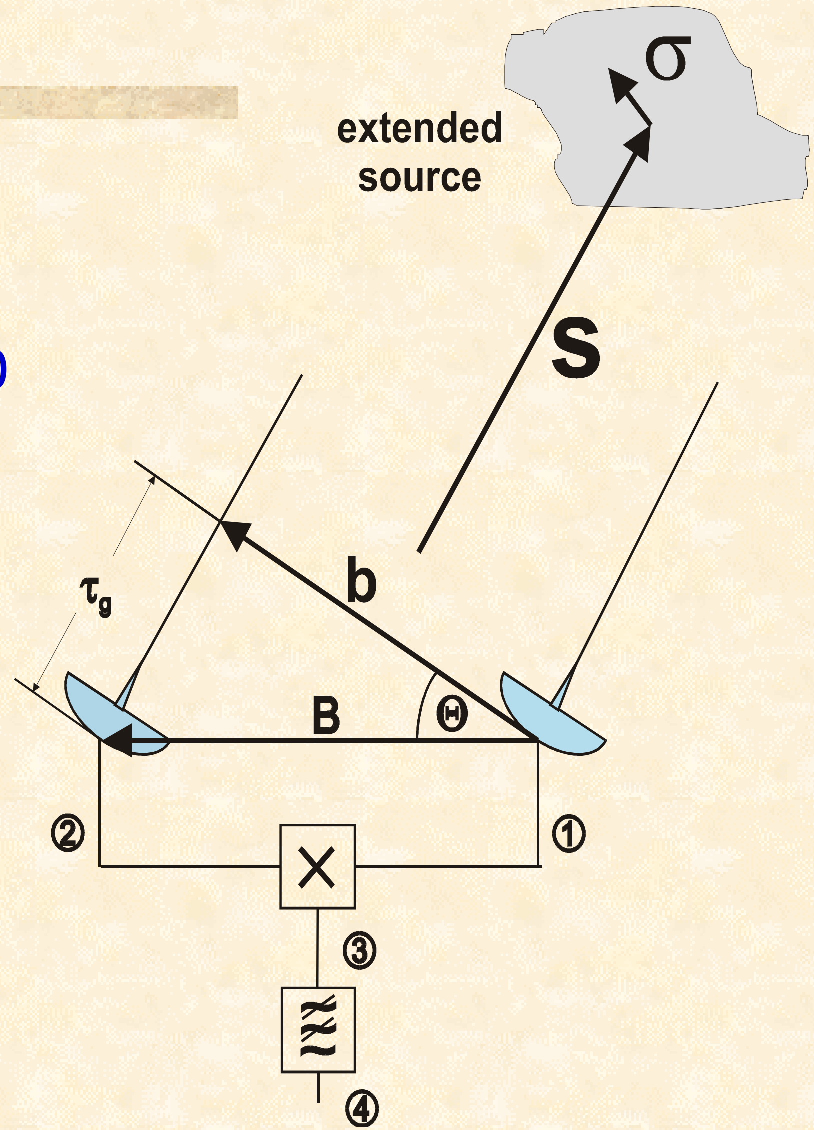
Extended Sources I

- point source = unrealistic case
- extended source = <u>sum</u> of point sources
- response of the interferometer = <u>sum</u> of the responses to point sources



Extended Sources II

- using vector notation:
  - **s**(t) = source vector (to source center)
  - **σ** = element deviation from source center
  - **B** = vector notation of baseline
  - **b** = projected spacing



Extended Sources III

$$\mathrm{R(t)} \propto \int_{-\infty}^{+\infty} \mathrm{d}\boldsymbol{\sigma} \cdot I(\boldsymbol{\sigma}) \cdot \cos\left(2\pi \mathbf{B} \cdot (\mathbf{s}(\mathrm{t}) + \boldsymbol{\sigma})\right)$$

$$= \mathrm{V} \cdot \exp\{i 2\pi \mathbf{B} \cdot \mathbf{s}(t)\}$$

$$\mathrm{V} = \int_{-\infty}^{+\infty} \mathrm{d}\boldsymbol{\sigma} \cdot I(\boldsymbol{\sigma}) \cdot \exp\{i 2\pi \mathbf{b} \cdot \boldsymbol{\sigma}\}$$

V = Visibility function = **Fourier transformation of source's brightness distribution**

Bandwidth Effects I

- wide bandwidths desirable $\rightarrow$ increasing S/N ratio

$$R(t) = \tfrac{1}{2}E^2 \cdot \cos\left(\frac{2\pi B \cdot \Theta(t)}{\lambda}\right)$$

from monochromatic case becomes:

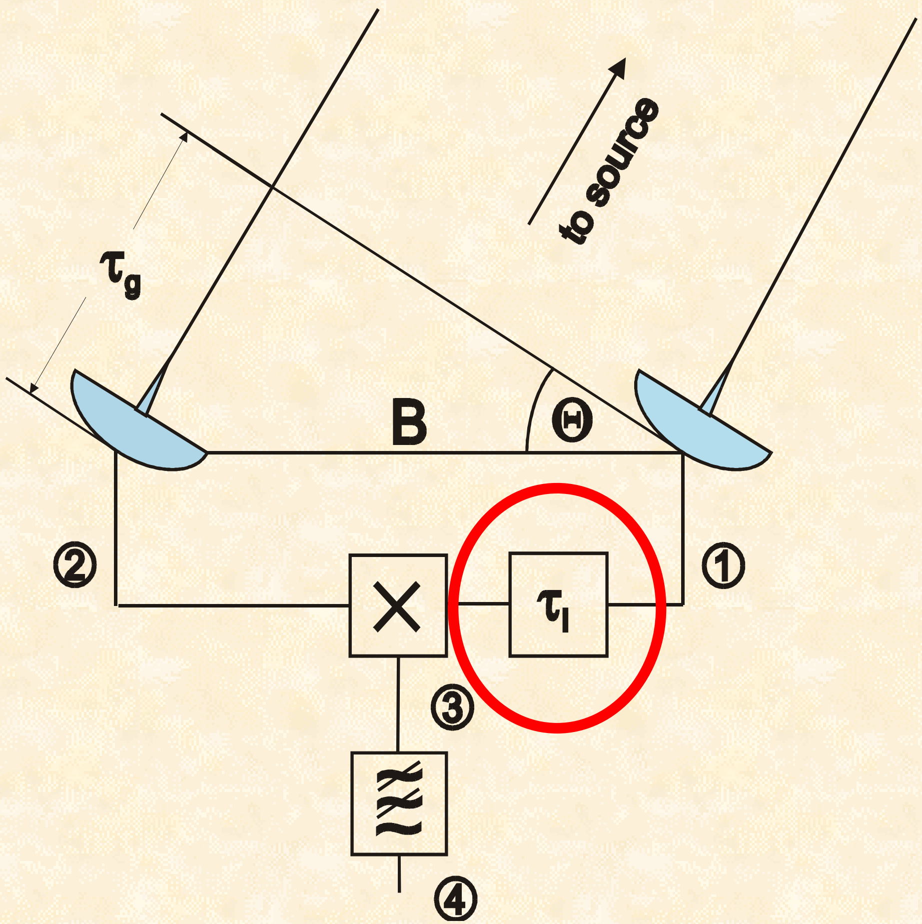
$$R(t) = \tfrac{1}{2}E^2 \cdot \int_{\omega_0}^{\omega_0+\Delta\omega} d(\omega) \cdot \alpha(\omega) \cdot \cos\left(\frac{2\pi B \cdot \Theta(t)}{\lambda}\right)$$

$\alpha(\omega)$ = frequency characteristics of equipment

- big $\Delta\omega$: τ_g can lead to loss of correlation **!**

Bandwidth Effects II

- instrumental delay τ_i for compensation
- exact compensation of τ_g only for **s**
- interferences from source elements at **s**+σ



Aperture Synthesis I

- two-element interferometer
  - $\longrightarrow$ one Fourier component of brightness distribution
  - $\longrightarrow$ measuring at one discrete spatial frequency
- spatial frequency
  - $\longrightarrow$ describes how fast the brightness changes with the direction (angle) of observation
  - $\longrightarrow$ analogy: frequencies describe how fast the amplitude of an electrical signal changes in time

Aperture Synthesis II

- single radio telescope = low-pass for spatial frequencies
- interferometer = band-pass for spatial frequencies
- reconstruction of brightness distribution needs measuring at many different spatial frequencies
- analogy: voice signal on a phone line
  - one measuring system measures at 1kHz
  - others at 300, 350, ... , 2800, 2900, 3000Hz
  - all together $\rightarrow$ approximation of the original signal
  - ordinary telephone $\cong$ filled aperture telescope

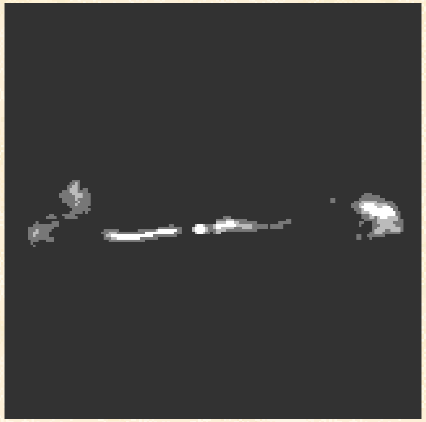
Aperture Synthesis III

Image reconstruction:

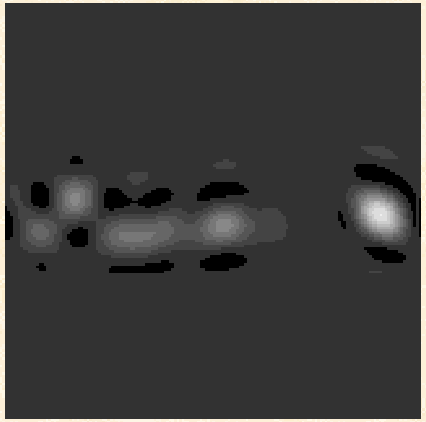
- inverse Fourier transformation of a set of Visibility functions
- cleaning of images necessary
  - CLEAN algorithm (Högbom 1974)
    - variants: Clark, Cotton-Schwab
  - Maximum Entropy Method (Wernecke 1975)

Aperture Synthesis IV

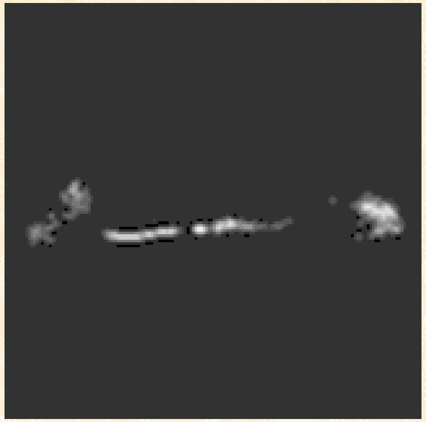
- image quality depends on number of elements



source's brightness distribution



9 x 9 components



40 x 40 components

VLBI

- first interferometers: directly connected by RF lines
- not feasible with greater distances
- **V**ery **L**ong **B**aseline **I**nterferometry:
  - inter-continental distances (baselines > 10000km)
  - synchronization of LOs: atomic frequency standards
  - correlation of signals: off-line
  - earth rotation $\longrightarrow$ different visibility functions with the same telescopes

Intensity Interferometer

- Hanbury Brown, Twiss (1968)
- interferometer without phase stable system (incoherent LOs)
- post-detection multiplication
- some correlation due to intensity fluctuatios
- low signal-to-noise ratio

Amateur Radio Astronomy and Interferometry I

- lots of amateur radio telescopes available
- main problem: LO and time synchronization
- ALLBIN project:
  - synchronization of time by Astra satellite
  - Phase 1: “only” accumulation of data
  - Phase 2: intensity interferometer (no LO synchronization necessary)
  - Phase 3: ALLBIN a “normal” interferometer ???

Amateur Radio Astronomy and Interferometry II

- What's missing to make ALLBIN a real interferometer? (Assume data exchange and communication from phase 1 working!)
  - phase 2:
    - understand Intensity Interferometer
    - write software to combine data from telescopes and for imaging
  - phase 3:
    - develop techniques to use satellite signal for LO
    - software (use as much as possible from first step)

Literature I

- Books
  - John D. Kraus, *Radio Astronomy*, 2^{nd} Edition, Cygnus-Quasar Books
  - Burke, Graham-Smith, *An Introduction to Radio Astronomy*, Cambridge Univ. Press 1997
  - Verschuur, Kellermann (Ed.), *Galactic and Extra-Galactic Radio Astronomy*, Chapter 10 by E.B.Fomalont & Melwyn C.H.Wright, Springer 1974
  - Kristen Rohlfs, *Tools of Radio Astronomy*, Chapter 6, Springer 1986
  - R.Wohlleben, H.Mattes, *Interferometrie in Radioastronomie und Radartechnik*, Vogel-Verlag 1973
- Articles
  - E.B.Fomalont, *Earth-Rotation Aperture Synthesis*, IEEE Proceedings Vol.61, No.9, 1973
  - A.R.Thompson, R.N.Bracewell, *Interpolation and Fourier Transformation of Fringe Visibilities*, Astronomical Journal, 1974
  - M.A.Cohen, *High-Resolution Observations of Radio Sources*, ARAA Vol. 7, 1969
  - M.Ishiguro, *Phase Error Correction in Multi-Element Radio Interferometers by Data Processing*, Astron.Astrophys.Supplement 15, 1974

Literature II

- Articles (cont'd)
  - S.I.Wernecke, *Two-dimensional Maximum Entropy Reconstruction of Radio Brightness*, Radio Science, Vol. 12, No. 5, 1977
- Internet
  - http://www.treasure-troves.com/physics/IntensityInterferometer.html
  - http://www.treasure-troves.com/physics/SynthesisImaging.html
  - http://www.treasure-troves.com/physics/Visibility.html
  - MERLIN User's Guide:
    http://www.jb.man.ac.uk/merlin/user_guide//preface.html
  - image cleaning algorithms:
    http://www.cv.nrao.edu/~abridle/deconvol/node7.html
  - Astronomical Optical Interferometry, A Literature Review by Bob Tubbs:
    http://www.cus.cam.ac.uk/~mt20/interferometry/ast_opt_int/page1.html